1.6 Piecewise Functions

YOU WILL NEED

- graph paper
- graphing calculator

GOAL

Understand, interpret, and graph situations that are described by piecewise functions.

LEARN ABOUT the Math

A city parking lot uses the following rules to calculate parking fees:

- A flat rate of \$5.00 for any amount of time up to and including the first hour
- A flat rate of \$12.50 for any amount of time over 1 h and up to and including 2 h
- A flat rate of \$13 plus \$3 per hour for each hour after 2 h

? How can you describe the function for parking fees in terms of the number of hours parked?

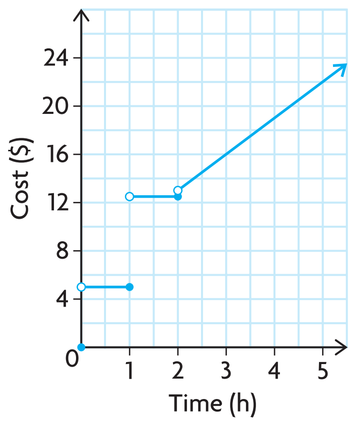
EXAMPLE 1 Representing the problem using a graphical model

Use a graphical model to represent the function for parking fees.

Solution

Time (h)	Parking Fee (\$)
0	0
0.25	5.00
0.50	5.00
1.00	5.00
1.25	12.50
1.50	12.50
2.00	12.50
2.50	14.50
3.00	16.00
4.00	19.00

Create a table of values.



Plot the points in the table of values. Use a solid dot to include a value in an interval. Use an open dot to exclude a value from an interval.

There is a solid dot at (0, 0) and an open dot at (0, 5) because the parking fee at 0 h is \$0.00.

There is a closed dot at (1, 5) and an open dot at (1, 12.50) because the parking fee at 1 h is \$5.00.

There is a closed dot at (2, 12.50) and an open dot at (2, 13) because the parking fee at 2 h is \$12.50.

The last part of the graph continues in a straight line since the rate of change is constant after 2 h.

The domain of this piecewise function is $x \geq 0$.

The function is linear over the domain, but it is discontinuous at $x = 0, 1,$ and 2.

piecewise function
a function defined by using two or more rules on two or more intervals; as a result, the graph is made up of two or more pieces of similar or different functions

Each part of a piecewise function can be described using a specific equation for the interval of the domain.

EXAMPLE 2 Representing the problem using an algebraic model

Use an algebraic model to represent the function for parking fees.

Solution

$y_1 = 0$ if $x = 0$

$y_2 = 5$ if $0 < x \leq 1$

$y_3 = 12.50$ if $1 < x \leq 2$

$y_4 = 3x + 13$ if $x > 2$

Write the relation for each rule.

$$f(x) = \begin{cases} 0, \text{ if } x = 0 \\ 5, \text{ if } 0 < x \leq 1 \\ 12.50, \text{ if } 1 < x \leq 2 \\ 3x + 13, \text{ if } x > 2 \end{cases}$$

Combine the relations into a piecewise function.

The domain of the function is $x \geq 0$.

The function is discontinuous at $x = 0$, 1, and 2 because there is a break in the function at each of these points.

Reflecting

A. How do you sketch the graph of a piecewise function?

B. How do you create the algebraic representation of a piecewise function?

C. How do you determine from a graph or from the algebraic representation of a piecewise function if there are any discontinuities?

APPLY the Math

EXAMPLE 3 Representing a piecewise function using a graph

Graph the following piecewise function.

$$f(x) = \begin{cases} x^2, \text{ if } x < 2 \\ 2x + 3, \text{ if } x \geq 2 \end{cases}$$

Solution

Create a table of values.

$f(x) = x^2$

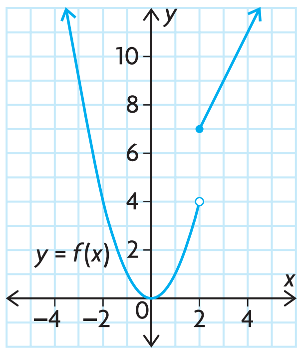
x	*f(x)*
−2	4
−1	1
0	0
1	1
2	4

$f(x) = 2x + 3$

x	*f(x)*
2	7
3	9
4	11
5	13
6	15

From the equations given, the graph consists of part of a parabola that opens up and a line that rises from left to right.

Both tables include $x = 2$ since this is where the description of the function changes.



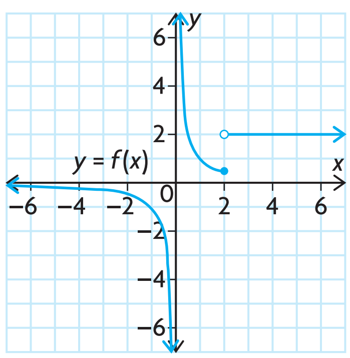
Plot the points, and draw the graph.

A solid dot is placed at (2, 7) since $x = 2$ is included with $f(x) = 2x + 3$. An open dot is placed at (2, 4) since $x = 2$ is excluded from $f(x) = x^2$.

$f(x)$ is discontinuous at $x = 2$.

EXAMPLE 4 Representing a piecewise function using an algebraic model

Determine the algebraic representation of the following piecewise function.



Solution

$$f(x) = \begin{cases} \dfrac{1}{x}, & \text{if } x \leq 2 \\ 2, & \text{if } x > 2 \end{cases}$$

The graph is made up of two pieces. One piece is part of the reciprocal function defined by $y = \frac{1}{x}$ when $x \leq 2$. The other piece is a horizontal line defined by $y = 2$ when $x > 2$. The solid dot indicates that point $\left(2, \frac{1}{2}\right)$ belongs with the reciprocal function.

EXAMPLE 5 Reasoning about the continuity of a piecewise function

Is this function continuous at the points where it is pieced together? Explain.

$$g(x) = \begin{cases} x + 1, \text{ if } x \leq 0 \\ 2x + 1, \text{ if } 0 < x < 3 \\ 4 - x^2, \text{ if } x \geq 3 \end{cases}$$

Solution

The graph is made up of three pieces. One piece is part of an increasing line defined by $y = x + 1$ when $x \leq 0$. The second piece is an increasing line defined by $y = 2x + 1$ when $0 < x < 3$. The third piece is part of a parabola that opens down, defined by $y = 4 - x^2$ when $x \geq 3$.

The function is continuous at the points where it is pieced together if the functions being joined have the same y-values at these points.

Calculate the values of the function at $x = 0$ using the relevant equations:

$$\begin{array}{ll} y = x + 1 & y = 2x + 1 \\ y = 0 + 1 & y = 2(0) + 1 \\ y = 1 & y = 1 \end{array}$$

The two y-values are the same, so the two linear pieces join each other at $x = 0$.

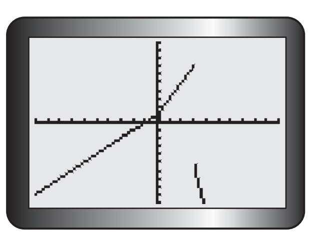
Calculate the values of the function at $x = 3$ using the relevant equations:

$$\begin{array}{ll} y = 2x + 1 & y = 4 - x^2 \\ y = 2(3) + 1 & y = 4 - 3^2 \\ y = 7 & y = -5 \end{array}$$

The two y-values are different, so the second linear piece does not join with the parabola at $x = 3$.

The function is discontinuous, since there is a break in the graph at $x = 3$.

Verify by graphing.



Tech Support

For help using a graphing calculator to graph a piecewise function, see Technical Appendix, T-16.

In Summary

Key Ideas

- Some functions are represented by two or more "pieces." These functions are called piecewise functions.
- Each piece of a piecewise function is defined for a specific interval in the domain of the function.

Need to Know

- To graph a piecewise function, graph each piece of the function over the given interval.
- A piecewise function can be either continuous or not. If all the pieces of the function join together at the endpoints of the given intervals, then the function is continuous. Otherwise, it is discontinuous at these values of the domain.

CHECK Your Understanding

1. Graph each piecewise function.

a) $f(x) = \begin{cases} 2, \text{ if } x < 1 \\ 3x, \text{ if } x \geq 1 \end{cases}$

b) $f(x) = \begin{cases} -2x, \text{ if } x < 0 \\ x + 4, \text{ if } x \geq 0 \end{cases}$

c) $f(x) = \begin{cases} |x|, \text{ if } x \leq -2 \\ -x^2, \text{ if } x > -2 \end{cases}$

d) $f(x) = \begin{cases} |x + 2|, \text{ if } x \leq -1 \\ -x^2 + 2, \text{ if } x > -1 \end{cases}$

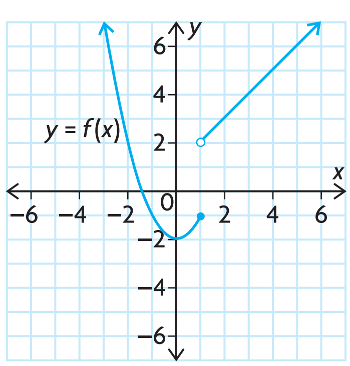
e) $f(x) = \begin{cases} \sqrt{x}, \text{ if } x < 4 \\ 2^x, \text{ if } x \geq 4 \end{cases}$

f) $f(x) = \begin{cases} \dfrac{1}{x}, \text{ if } x < 1 \\ -x, \text{ if } x \geq 1 \end{cases}$

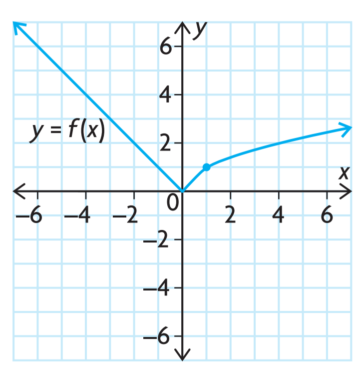
2. State whether each function in question 1 is continuous or not. If not, state where it is discontinuous.

3. Write the algebraic representation of each piecewise function, using function notation.

a)



b)



4. State the domain of each piecewise function in question 3, and comment on the continuity of the function.

PRACTISING

5. K Graph the following piecewise functions. Determine whether each function is continuous or not, and state the domain and range of the function.

a) $f(x) = \begin{cases} 2, \text{ if } x < -1 \\ 3, \text{ if } x \geq -1 \end{cases}$

b) $f(x) = \begin{cases} -x, \text{ if } x \leq 0 \\ x, \text{ if } x > 0 \end{cases}$

c) $f(x) = \begin{cases} x^2 + 1, \text{ if } x < 2 \\ 2x + 1, \text{ if } x \geq 2 \end{cases}$

d) $f(x) = \begin{cases} 1, \text{ if } x < -1 \\ x + 2, \text{ if } -1 \leq x \leq 3 \\ 5, \text{ if } x > 3 \end{cases}$

6. A Graham's long-distance telephone plan includes the first 500 min per month in the \$15.00 monthly charge. For each minute after 500 min, Graham is charged \$0.02. Write a function that describes Graham's total long-distance charge in terms of the number of long distance minutes he uses in a month.

7. Many income tax systems are calculated using a tiered method. Under a certain tax law, the first \$100 000 of earnings are subject to a 35% tax; earnings greater than \$100 000 and up to \$500 000 are subject to a 45% tax. Any earnings greater than \$500 000 are taxed at 55%. Write a piecewise function that models this situation.

8. T Find the value of k that makes the following function continuous. Graph the function.

$$f(x) = \begin{cases} x^2 - k, \text{ if } x < -1 \\ 2x - 1, \text{ if } x \geq -1 \end{cases}$$

9. The fish population, in thousands, in a lake at any time, x, in years is modelled by the following function:

$$f(x) = \begin{cases} 2^x, \text{ if } 0 \leq x \leq 6 \\ 4x + 8, \text{ if } x > 6 \end{cases}$$

This function describes a sudden change in the population at time $x = 6$, due to a chemical spill.

a) Sketch the graph of the piecewise function.

b) Describe the continuity of the function.

c) How many fish were killed by the chemical spill?

d) At what time did the population recover to the level it was before the chemical spill?

e) Describe other events relating to fish populations in a lake that might result in piecewise functions.

10. Create a flow chart that describes how to plot a piecewise function with two pieces. In your flow chart, include how to determine where the function is continuous.

11. **C** An absolute value function can be written as a piecewise function that involves two linear functions. Write the function $f(x) = |x + 3|$ as a piecewise function, and graph your piecewise function to check it.

12. The demand for a new CD is described by

$$D(p) = \begin{cases} \frac{1}{p^2}, \text{ if } 0 < p \leq 15 \\ 0, \text{ if } p > 15 \end{cases}$$

where D is the demand for the CD at price p, in dollars. Determine where the demand function is discontinuous and continuous.

Extending

13. Consider a function, $f(x)$, that takes an element of its domain and rounds it down to the nearest 10. Thus, $f(15.6) = 10$, while $f(21.7) = 20$ and $f(30) = 30$. Draw the graph, and write the piecewise function. You may limit the domain to $x \in [0, 50)$. Why do you think graphs like this one are often referred to as *step functions*?

14. Explain why there is no value of k that will make the following function continuous.

$$f(x) = \begin{cases} 5x, \text{ if } x < -1 \\ x + k, \text{ if } -1 \leq x \leq 3 \\ 2x^2, \text{ if } x > 3 \end{cases}$$

15. The *greatest integer function* is a step function that is written as $f(x) = [x]$, where $f(x)$ is the greatest integer less than or equal to x. In other words, the greatest integer function rounds any number down to the nearest integer. For example, the greatest integer less than or equal to the number [5.3] is 5, while the greatest integer less than or equal to the number $[-5.3]$ is -6. Sketch the graph of $f(x) = [x]$.

16. **a)** Create your own piecewise function using three different transformed parent functions.

b) Graph the function you created in part a).

c) Is the function you created continuous or not? Explain.

d) If the function you created is not continuous, change the interval or adjust the transformations used as required to change it to a continuous function.

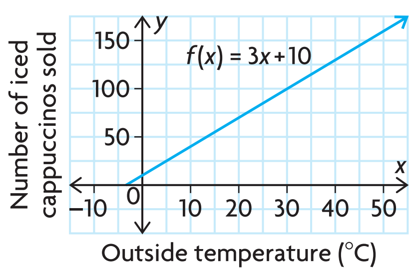
1.7 Exploring Operations with Functions

GOAL

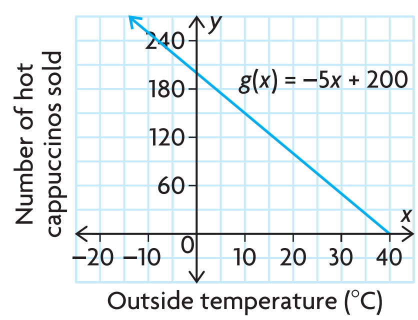
Explore the properties of the sum, difference, and product of two functions.

A popular coffee house sells iced cappuccino for \$4 and hot cappuccino for \$3. The manager would like to predict the relationship between the outside temperature and the total daily revenue from each type of cappuccino sold. The manager discovers that every 1 °C increase in temperature leads to an increase in the sales of cold drinks by three cups per day and to a decrease in the sales of hot drinks by five cups per day.

The function $f(x) = 3x + 10$ can be used to model the number of iced cappuccinos sold.



The function $g(x) = -5x + 200$ can be used to model the number of hot cappuccinos sold.



In both functions, x represents the daily average outside temperature. In the first function, $f(x)$ represents the daily average number of iced cappuccinos sold. In the second function, $g(x)$ represents the daily average number of hot cappuccinos sold.

? How does the outside temperature affect the daily revenue from cappuccinos sold?

A. Make a table of values for each function, with the temperature in intervals of 5°, from 0° to 40°.

B. What does $h(x) = f(x) + g(x)$ represent?

C. Simplify $h(x) = (3x + 10) + (-5x + 200)$.

D. Make a table of values for the function in part C, with the temperature in intervals of 5°, from 0° to 40°. How do the values compare with the values in each table you made in part A? How do the domains of $f(x)$, $g(x)$, and $h(x)$ compare?

E. What does $h(x) = f(x) - g(x)$ represent?

F. Simplify $h(x) = (3x + 10) - (-5x + 200)$.

G. Make a table of values for the function in part F, with the temperature in intervals of 5°, from 0° to 40°. How do the values compare with the values in each table you made in part A? How do the domains of $f(x)$, $g(x)$, and $h(x)$ compare?

H. What does $R(x) = 4f(x) + 3g(x)$ represent?

I. Simplify $R(x) = 4(3x + 10) + 3(-5x + 200)$.

J. Make a table of values for the function in part I, with the temperature in intervals of 5°, from 0° to 40°. How do the values compare with the values in each table you made in part A? How do the domains of $f(x)$, $g(x)$, and $R(x)$ compare?

K. How does temperature affect the daily revenue from cappuccinos sold?

Reflecting

L. Explain how the sum function, $h(x)$, would be different if
a) both $f(x)$ and $g(x)$ were increasing functions
b) both $f(x)$ and $g(x)$ were decreasing functions

M. What does the function $k(x) = g(x) - f(x)$ represent? Is its graph identical to the graph of $h(x) = f(x) - g(x)$? Explain.

N. Determine the function $h(x) = f(x) \times g(x)$. Does this function have any meaning in the context of the daily revenue from cappuccinos sold? Explain how the table of values for this function is related to the tables of values you made in part A.

O. If you are given the graphs of two functions, explain how you could create a graph that represents
a) the sum of the two functions
b) the difference between the two functions
c) the product of the two functions

In Summary

Key Idea

- If two functions have domains that overlap, they can be added, subtracted, or multiplied to create a new function on that shared domain.

Need to Know

- Two functions can be added, subtracted, or multiplied graphically by adding, subtracting, or multiplying the values of the dependent variable for identical values of the independent variable.
- Two functions can be added, subtracted, or multiplied algebraically by adding, subtracting, or multiplying the expressions for the dependent variable and then simplifying.
- The properties of each original function have an impact on the properties of the new function.

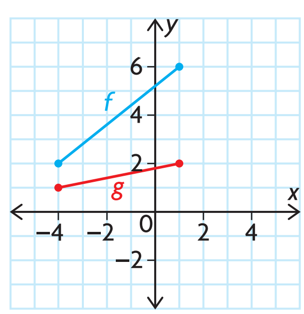
FURTHER Your Understanding

1. Let $f = \{(-4, 4), (-2, 4), (1, 3), (3, 5), (4, 6)\}$ and $g = \{(-4, 2), (-2, 1), (0, 2), (1, 2), (2, 2), (4, 4)\}$. Determine:

a) $f + g$ **b)** $f - g$ **c)** $g - f$ **d)** fg

2. Use the graphs of f and g to sketch the graphs of $f + g$.

a)



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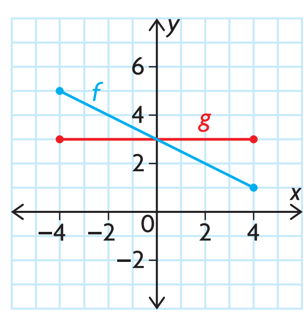
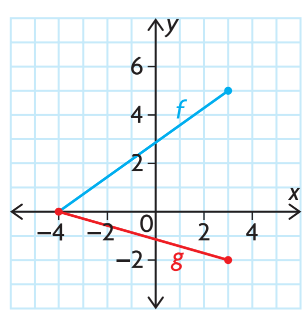


3. Use the graphs of f and g to sketch the graphs of $f - g$.

a)



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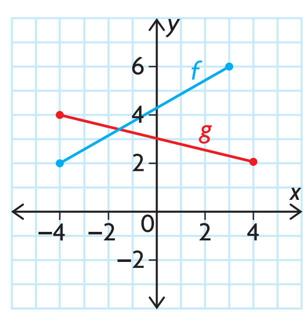
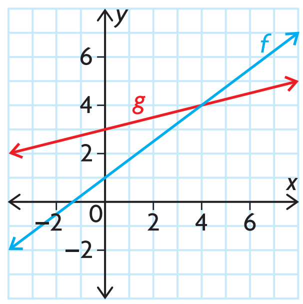


4. Use the graphs of f and g to sketch the graphs of fg.

a)



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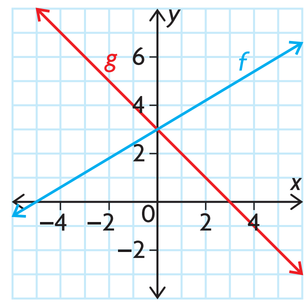


5. Determine the equation of each new function, and then sketch its graph.
   a) $h(x) = f(x) + g(x)$, where $f(x) = x^2$ and $g(x) = -x^2$
   b) $p(x) = m(x) - n(x)$, where $m(x) = x^2$ and $n(x) = -7x + 12$
   c) $r(x) = s(x) + t(x)$, where $s(x) = |x|$ and $t(x) = 2^x$
   d) $a(x) = b(x) \times c(x)$, where $b(x) = x$ and $c(x) = x^2$

6. a) Using the graphs you sketched in question 5, compare and contrast the relationship between the properties of the original functions and the properties of the new function.
   b) Which properties of the original functions determined the properties of the new function?

7. Let $f(x) = x + 3$ and $g(x) = -x^2 + 5$, $x \in \mathbf{R}$.
   a) Sketch each graph on the same set of axes.
   b) Make a table of values for $-3 \le x \le 3$, and determine the corresponding values of $h(x) = f(x) \times g(x)$.
   c) Use the table to sketch $h(x)$ on the same axes. Describe the shape of the graph.
   d) Determine the algebraic model for $h(x)$. What is its degree?
   e) What is the domain of $h(x)$? How does this domain compare with the domains of $f(x)$ and $g(x)$?

8. Let $f(x) = x^2 + 2$ and $g(x) = x^2 - 2$, $x \in \mathbf{R}$.
   a) Sketch each graph on the same set of axes.
   b) Make a table of values for $-3 \le x \le 3$, and determine the corresponding values of $h(x) = f(x) \times g(x)$.
   c) Use the table to sketch $h(x)$ on the same axes. Describe the shape of the graph.
   d) Determine the algebraic model for $h(x)$. What is its degree?
   e) What is the domain of $h(x)$?

1 Chapter Review

FREQUENTLY ASKED Questions

Study Aid

- See Lesson 1.4, Examples 2, 3, and 4.
- Try Chapter Review Questions 7, 8, and 9.

Q: In what order are transformations performed on a function?

Function → Perform all stretches/compressions and reflections. → Perform all translations.

A: All stretches/compressions (vertical and horizontal) and reflections can be applied at the same time by multiplying the *x*- and *y*-coordinates on the parent function by the appropriate factors. Both vertical and horizontal translations can then be applied by adding or subtracting the relevant numbers to the *x*- and *y*-coordinates of the points.

Study Aid

- See Lesson 1.5, Examples 1, 2, and 3.
- Try Chapter Review Questions 10 to 13.

Q: How do you find the inverse relation of a function?

A: You can find the inverse relation of a function numerically, graphically, or algebraically.

To find the inverse relation of a function numerically, using a table of values, switch the values for the independent and dependent variables.

$f(x)$	f^{-1}
(x, y)	(y, x)

To find the inverse relation graphically, reflect the graph of the function in the line $y = x$. This is accomplished by switching the *x*- and *y*-coordinates in each ordered pair.

To find the algebraic representation of the inverse relation, interchange the positions of the *x*- and *y*-variables in the function and solve for *y*.

Q: Is an inverse of a function always a function?

A: No; if an element in the domain of the original function corresponds to more than one number in the range, then the inverse relation is not a function.

Study Aid

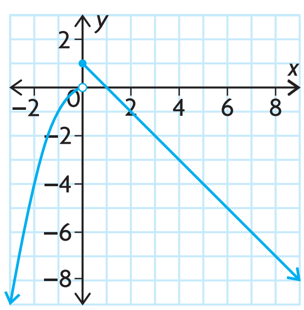
- See Lesson 1.5, Examples 1, 2, and 3.
- Try Chapter Review Questions 10 to 13.

Q: What is a piecewise function?

A: A piecewise function is a function that has two or more function rules for different parts of its domain.

For example, the function defined by $f(x) = \begin{cases} -x^2, \text{ if } x < 0 \\ -x + 1, \text{ if } x \geq 0 \end{cases}$

consists of two pieces. The first equation defines half of a parabola that opens down when $x < 0$. The second equation defines a decreasing line with a y-intercept of 1 when $x \geq 0$. The graph confirms this.



Study Aid

- See Lesson 1.6, Examples 1, 2, 3, and 4.
- Try Chapter Review Questions 14 to 17.

Q: If you are given the graphs or equations of two functions, how can you create a new function?

A: You can create a new function by adding, subtracting, or multiplying the two given functions.

This can be done graphically by adding, subtracting, or multiplying the y-coordinates in each pair of ordered pairs that have identical x-coordinates.

This can be done algebraically by adding, subtracting, or multiplying the expressions for the dependent variable and then simplifying.

Study Aid

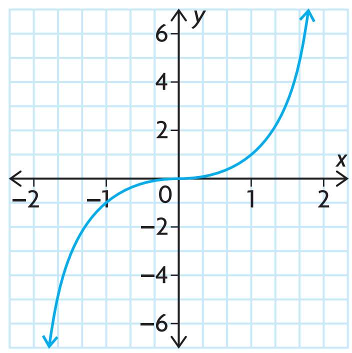
- See Lesson 1.7.
- Try Chapter Review Questions 18 to 21.

PRACTICE Questions

Lesson 1.1

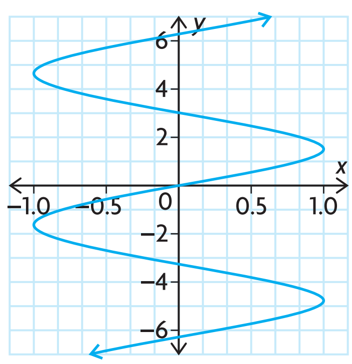
1. Determine whether each relation is a function, and state its domain and range.

a)



b) $3x^2 + 2y = 6$

c)



d) $x = 2^y$

2. A cell phone company charges a monthly fee of \$30, plus \$0.02 per minute of call time.

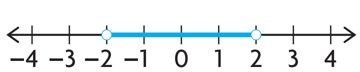
a) Write the monthly cost function, $C(t)$, where t is the amount of time in minutes of call time during a month.

b) Find the domain and range of C.

Lesson 1.2

3. Graph $f(x) = 2|x + 3| - 1$, and state the domain and range.

4. Describe this interval using absolute value notation.



Lesson 1.3

5. For each pair of functions, give a characteristic that the two functions have in common and a characteristic that distinguishes them.

a) $f(x) = x^2$ and $g(x) = \sin x$

b) $f(x) = \frac{1}{x}$ and $g(x) = x$

c) $f(x) = |x|$ and $g(x) = x^2$

d) $f(x) = 2^x$ and $g(x) = x$

6. Identify the intervals of increase/decrease, the symmetry, and the domain and range of each function.

a) $f(x) = 3x$

b) $f(x) = x^2 + 2$

c) $f(x) = 2^x - 1$

Lesson 1.4

7. For each of the following equations, state the parent function and the transformations that were applied. Graph the transformed function.

a) $y = |x + 1|$

b) $y = -0.25\sqrt{3(x + 7)}$

c) $y = -2\sin(3x) + 1,\ 0 \le x \le 360°$

d) $y = 2^{-2x} - 3$

8. The graph of $y = x^2$ is horizontally stretched by a factor of 2, reflected in the x-axis, and shifted 3 units down. Find the equation that results from the transformation, and graph it.

9. (2, 1) is a point on the graph of $y = f(x)$. Find the corresponding point on the graph of each of the following functions.

a) $y = -f(-x) + 2$

b) $y = f(-2(x + 9)) - 7$

c) $y = f(x - 2) + 2$

d) $y = 0.3f(5(x - 3))$

e) $y = 1 - f(1 - x)$

f) $y = -f(2(x - 8))$

Lesson 1.5

10. For each point on a function, state the corresponding point on the inverse relation.

a) $(1, 2)$ **b)** $(-1, -9)$ **c)** $(0, 7)$ **d)** $f(5) = 7$ **e)** $g(0) = -3$ **f)** $h(1) = 10$

11. Given the domain and range of a function, state the domain and range of the inverse relation.

a) $D = \{x \in \mathbf{R}\}, R = \{y \in \mathbf{R} \mid -2 < y < 2\}$

b) $D = \{x \in \mathbf{R} \mid x \geq 7\}, R = \{y \in \mathbf{R} \mid y < 12\}$

12. Graph each function and its inverse relation on the same set of axes. Determine whether the inverse relation is a function.

a) $f(x) = x^2 - 4$ **b)** $g(x) = 2^x$

13. Find the inverse of each function.

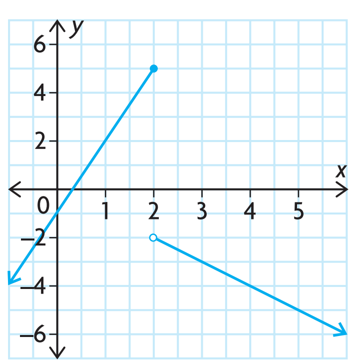
a) $f(x) = 2x + 1$ **b)** $g(x) = x^3$

Lesson 1.6

14. Graph the following function. Determine whether it is discontinuous and, if so, where. State the domain and the range of the function.

$$f(x) = \begin{cases} 2x, \text{ if } x < 1 \\ x + 1, \text{ if } x \geq 1 \end{cases}$$

15. Write the algebraic representation for the following piecewise function, using function notation.



16. If $f(x) = \begin{cases} x^2 + 1, \text{ if } x < 1 \\ 3x, \text{ if } x \geq 1 \end{cases}$

is $f(x)$ continuous at $x = 1$? Explain.

17. A telephone company charges \$30 a month and gives the customer 200 free call minutes. After the 200 min, the company charges \$0.03 a minute.

a) Write the function using function notation.

b) Find the cost for talking 350 min in a month.

c) Find the cost for talking 180 min in a month.

Lesson 1.7

18. Given $f = \{(0, 6), (1, 3), (4, 7), (5, 8)\}$ and $g = \{(-1, 2), (1, 4), (2, 3), (4, 8), (8, 9)\}$, determine the following.

a) $f(x) + g(x)$

b) $f(x) - g(x)$

c) $[f(x)][g(x)]$

19. Given $f(x) = 2x^2 - 2x, -2 \leq x \leq 3$ and $g(x) = -4x, -3 \leq x \leq 5$, graph the following.

a) f **b)** g **c)** $f + g$ **d)** $f - g$ **e)** fg

20. $f(x) = x^2 + 2x$ and $g(x) = x + 1$. Match the answer with the operation.

Answer:	Operation:
a) $x^3 + 3x^2 + 2x$	**A** $f(x) + g(x)$
b) $-x^2 - x + 1$	**B** $f(x) - g(x)$
c) $x^2 + 3x + 1$	**C** $g(x) - f(x)$
d) $x^2 + x - 1$	**D** $f(x) \times g(x)$

21. $f(x) = x^3 + 2x^2$ and $g(x) = -x + 6$,

a) Complete the table.

x	−3	−2	−1	0	1	2
$f(x)$						
$g(x)$						
$(f + g)(x)$						

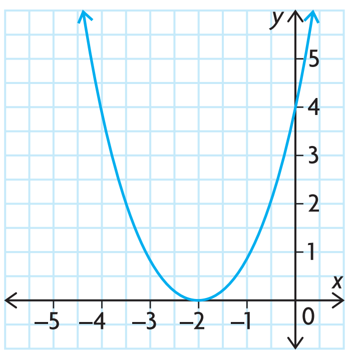
b) Use the table to graph $f(x)$ and $g(x)$ on the same axes.

c) Graph $(f + g)(x)$ on the same axes as part b).

d) State the equation of $(f + g)(x)$.

e) Verify the equation of $(f + g)(x)$ using two of the ordered pairs in the table.

Chapter Self-Test



1. Consider the graph of the relation shown.
   a) Is the relation a function? Explain.
   b) State the domain and range.

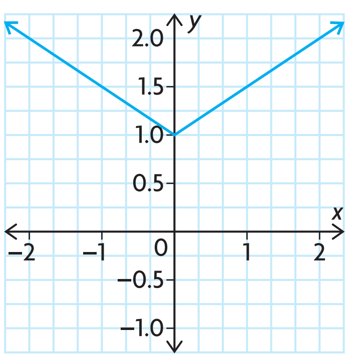
2. Given the following information about a function:
- $D = \{x \in \mathbf{R}\}$
- $R = \{y \in \mathbf{R} \mid y \geq -2\}$
- decreasing on the interval $(-\infty, 0)$
- increasing on the interval $(0, \infty)$

   a) What is a possible parent function?
   b) Draw a possible graph of the function.
   c) Describe the transformation that was performed.

3. Show algebraically that the function $f(x) = |3x| + x^2$ is an even function.

4. Both $f(x) = x^2$ and $g(x) = 2^x$ have a domain of all real numbers. List as many characteristics as you can to distinguish the two functions.

5. Describe the transformations that must be applied to $y = x^2$ to obtain the function $f(x) = -(x + 3)^2 - 5$, then graph the function.



6. Given the graph shown, describe the transformations that were performed to get this function. Write the algebraic representation, using function notation.

7. $(3, 5)$ is a point on the graph of $y = f(x)$. Find the corresponding point on the graph of each of the following relations.
   a) $y = 3f(-x + 1) + 2$ **b)** $y = f^{-1}(x)$

8. Find the inverse of $f(x) = -2(x + 1)$.

9. A certain tax policy states that the first \$50 000 of income is taxed at 5% and any income above \$50 000 is taxed at 12%.
   a) Calculate the tax on \$125 000.
   b) Write a function that models the tax policy.

10. **a)** Sketch the graph of $f(x)$ where $f(x) = \begin{cases} 2^x + 1, \text{ if } x < 0 \\ \sqrt{x} + 3, \text{ if } x \geq 0 \end{cases}$
   b) Is $f(x)$ continuous over its entire domain? Explain.
   c) State the intervals of increase and decrease.
   d) State the domain and range of this function.

1 Chapter Task

Modelling with Functions

In 1950, a team of chemists led by Dr. W. F. Libby developed a method for determining the age of any natural specimen, up to approximately 60 000 years of age. Dr. Libby's method is based on the fact that all living materials contain traces of carbon-14. His method involves measuring the percent of carbon-14 that remains when a specimen is found.

The percent of carbon-14 that remains in a specimen after various numbers of years is shown in the table below.

Years	Carbon-14 Remaining (%)
5 730	50.0
11 460	25.0
17 190	12.5
22 920	6.25
28 650	3.125
34 380	1.5625

? How can you use the function $P(t) = 100(0.5)^{\frac{t}{5730}}$ to model this situation and determine the age of a natural specimen?

A. What percent of carbon is remaining for $t = 0$? What does this mean in the context of Dr. Libby's method?

B. Draw a graph of the function $P(t) = 100(0.5)^{\frac{t}{5730}}$, using the given table of values.

C. What is a reasonable domain for $P(t)$? What is a reasonable range?

D. Determine the approximate age of a specimen, given that $P(t) = 70$.

E. Draw the graph of the inverse function.

F. What information does the inverse function provide?

G. What are the domain and range of the inverse function?

Task *Checklist*

- ✔ Did you show all your steps?
- ✔ Did you draw and label your graphs accurately?
- ✔ Did you determine the age of the specimen that had 70% carbon-14 remaining?
- ✔ Did you explain your thinking clearly?

Answers

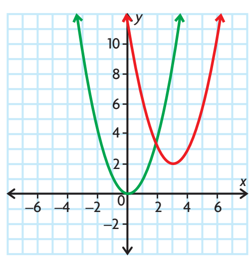
Chapter 1

Getting Started, p. 2

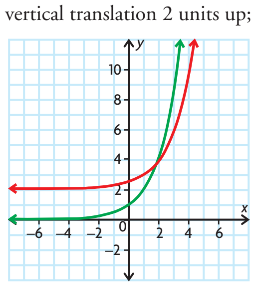
1. **a)** 6 **b)** -6 **c)** $-\frac{51}{16}$ **d)** $a^2 + 5a$

2. **a)** $(x + y)(x + y)$
b) $(5x - 1)(x - 3)$
c) $(x + y + 8)(x + y - 8)$
d) $(a + b)(x - y)$

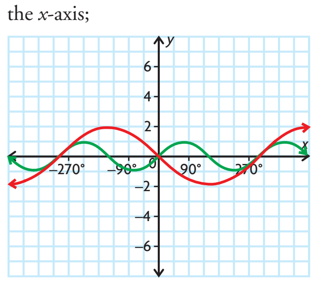
3. **a)** horizontal translation 3 units to the right, vertical translation 2 units up;



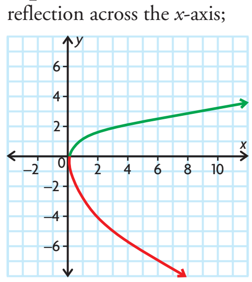
b) horizontal translation 1 unit to the right, vertical translation 2 units up;



c) horizontal stretch by a factor of 2, vertical stretch by a factor of 2, reflection across the x-axis;



d) horizontal compression by a factor of $\frac{1}{2}$, vertical stretch by a factor of 2, reflection across the x-axis;



4. **a)** $D = \{x \in \mathbf{R} \mid -2 \le x \le 2\}$, $R = \{y \in \mathbf{R} \mid 0 \le y \le 2\}$
b) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y \ge -19\}$
c) $D = \{x \in \mathbf{R} \mid x \ne 0\}$, $R = \{y \in \mathbf{R} \mid y \ne 0\}$
d) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid -3 \le y \ge 3\}$
e) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y > 0\}$

5. **a)** This is not a function; it does not pass the vertical line test.
b) This is a function; for each x-value, there is exactly one corresponding y-value.
c) This is not a function; for each x-value greater than 0, there are two corresponding y-values.
d) This is a function; for each x-value, there is exactly one corresponding y-value.
e) This is a function; for each x-value, there is exactly one corresponding y-value.

6. **a)** 8
b) about 2.71

7. If a relation is represented by a set of ordered pairs, a table, or an arrow diagram, one can determine if the relation is a function by checking that each value of the independent variable is paired with no more than one value of the dependent variable. If a relation is represented using a graph or scatter plot, the vertical line test can be used to determine if the relation is a function. A relation may also be represented by a description/rule or by using function notation or an equation. In these cases, one can use reasoning to determine if there is more than one value of the dependent variable paired with any value of the independent variable.

Lesson 1.1, pp. 11–13

1. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid -4 \le y \le -2\}$; This is a function because it passes the vertical line test.
b) $D = \{x \in \mathbf{R} \mid -1 \le x \le 7\}$; $R = \{y \in \mathbf{R} \mid -3 \le y \le 1\}$; This is a function because it passes the vertical line test.
c) $D = \{1, 2, 3, 4\}$; $R = \{-5, 4, 7, 9, 11\}$; This is not a function because 1 is sent to more than one element in the range.
d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$; This is a function because every element in the domain produces exactly one element in the range.
e) $D = \{-4, -3, 1, 2\}$; $R = \{0, 1, 2, 3\}$; This is a function because every element of the domain is sent to exactly one element in the range.
f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \le 0\}$; This is a function because every element in the domain produces exactly one element in the range.

2. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \le -3\}$; This is a function because every element in the domain produces exactly one element in the range.
b) $D = \{x \in \mathbf{R} \mid x \ne -3\}$; $R = \{y \in \mathbf{R} \mid y \ne 0\}$; This is a function because every element in the domain produces exactly one element in the range.
c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 0\}$; This is a function because every element in the domain produces exactly one element in the range.
d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid 0 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.
e) $D = \{x \in \mathbf{R} \mid -3 \le x \le 3\}$; $R = \{y \in \mathbf{R} \mid -3 \le y \le 3\}$; This is not a function because (0, 3) and (0, 3) are both in the relation.
f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid -2 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.

3. **a)** function; $D = \{1, 3, 5, 7\}$; $R = \{2, 4, 6\}$
b) function; $D = \{0, 1, 2, 5\}$; $R = \{-1, 3, 6\}$
c) function; $D = \{0, 1, 2, 3\}$; $R = \{2, 4\}$
d) not a function; $D = \{2, 6, 8\}$; $R = \{1, 3, 5, 7\}$
e) not a function; $D = \{1, 10, 100\}$; $R = \{0, 1, 2, 3\}$
f) function; $D = \{1, 2, 3, 4\}$; $R = \{1, 2, 3, 4\}$

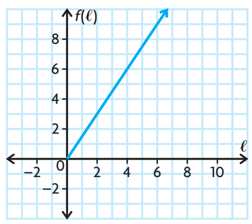
4. **a)** function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \ge 2\}$.
b) not a function; $D = \{x \in \mathbf{R} \mid x \ge 2\}$; $R = \{y \in \mathbf{R}\}$
c) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \ge -0.5\}$
d) not a function; $D = \{x \in \mathbf{R} \mid x \ge 0\}$; $R = \{y \in \mathbf{R}\}$
e) function; $D = \{x \in \mathbf{R} \mid x \ne 0\}$; $R = \{y \in \mathbf{R} \mid y \ne 0\}$
f) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

5. **a)** $y = x + 3$ **b)** $y = 2x - 5$ **c)** $y = 3(x - 2)$ **d)** $y = -x + 5$

6. **a)** The length is twice the width.

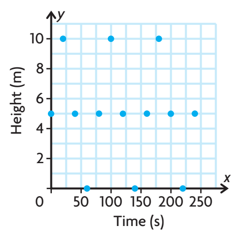
b) $f(l) = \frac{3}{2}l$

c)



d) length $= 8$ m; width $= 4$ m

7. **a)**

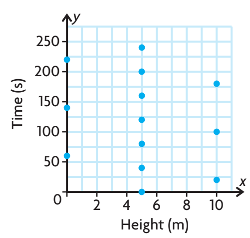


b) D $= \{0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240\}$

c) R $= \{0, 5, 10\}$

d) It is a function because it passes the vertical line test.

e)



f) It is not a function because (5, 0) and (5, 40) are both in the relation.

8. **a)** $\{(1, 2), (3, 4), (5, 6)\}$
b) $\{(1, 2), (3, 2), (5, 6)\}$
c) $\{(2, 1), (2, 3), (5, 6)\}$

9. If a vertical line passes through a function and hits two points, those two points have identical *x*-coordinates and different *y*-coordinates. This means that one *x*-coordinate is sent to two different elements in the range, violating the definition of *function*.

10. **a)** Yes, because the distance from (4, 3) to (0, 0) is 5.
b) No, because the distance from (1, 5) to (0, 0) is not 5.
c) No, because (4, 3) and $(4, -3)$ are both in the relation.

11. **a)** $g(x) = x^2 + 3$

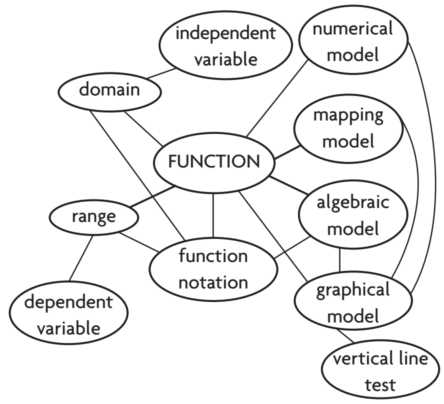
b) $g(3) - g(2) = 12 - 7$
$\qquad = 5$

$g(3 - 2) = g(1)$
$\qquad = 4$

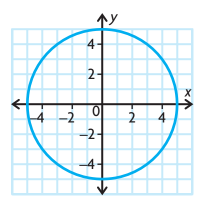
So, $g(3) - g(2) \neq g(3 - 2)$.

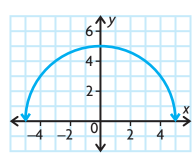
12. **a)** $f(6) = 12; f(7) = 8; f(8) = 15$
b) Yes, $f(15) = f(3) \times f(5)$
c) Yes, $f(12) = f(3) \times f(4)$
d) Yes, there are others that will work. $f(a) \times f(b) = f(a \times b)$ whenever a and b have no common factors other than 1.

13. Answers may vary. For example:



14.





The first is not a function because it fails the vertical line test:
D $= \{x \in \mathbf{R} \mid -5 \leq x \leq 5\}$;
R $= \{y \in \mathbf{R} \mid -5 \leq y \leq 5\}$.
The second is a function because it passes the vertical line test:
D $= \{x \in \mathbf{R} \mid -5 \leq x \leq 5\}$;
R $= \{y \in \mathbf{R} \mid 0 \leq y \leq 5\}$.

15. *x* is a function of *y* if the graph passes the horizontal line test. This occurs when any horizontal line hits the graph at most once.

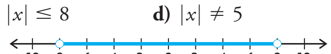
Lesson 1.2, p. 16

1. $|-5|, |12|, |-15|, |20|, |-25|$

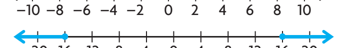
2. **a)** 22 **b)** -35 **c)** 18 **d)** 11 **e)** -2 **f)** -2

3. **a)** $|x| > 3$ **b)** $|x| \leq 8$ **c)** $|x| \geq 1$ **d)** $|x| \neq 5$

4. **a)**

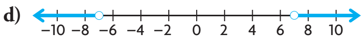


b)



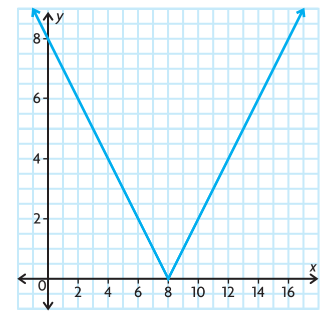
c) The absolute value of a number is always greater than or equal to 0. There are no solutions to this inequality.

d)



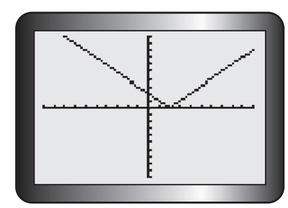
5. **a)** $|x| \leq 3$ **b)** $|x| > 2$ **c)** $|x| \geq 2$ **d)** $|x| < 4$

6.

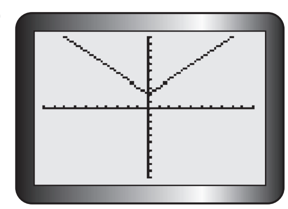


a) The graphs are the same.
b) Answers may vary. For example, $x - 8 = -(-x + 8)$, so they are negatives of each other and have the same absolute value.

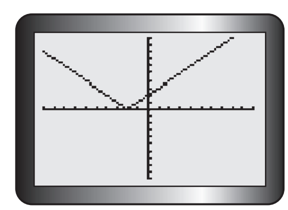
7. **a)**



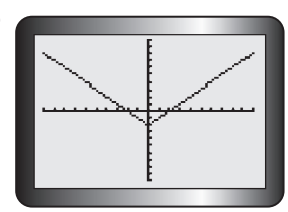
b)



c)



d)

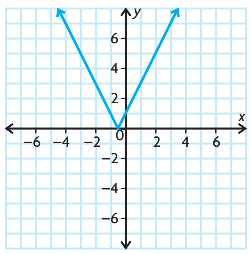


8. When the number you are adding or subtracting is inside the absolute value signs, it moves the function to the left (when adding) or to the right (when subtracting) of the origin. When the number you are

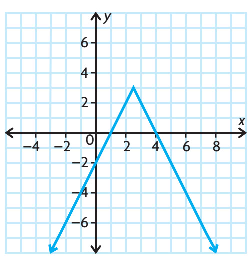
Answers

adding or subtracting is outside the absolute value signs, it moves the function down (when subtracting) or up (when adding) from the origin.
The graph of the function will be the absolute value function moved to the left 3 units and up 4 units from the origin.

9. This is the graph of $g(x) = |x|$ horizontally compressed by a factor of $\frac{1}{2}$ and translated $\frac{1}{2}$ unit to the left.



10. This is the graph of $g(x) = |x|$ horizontally compressed by a factor of $\frac{1}{2}$, reflected over the x-axis, translated $2\frac{1}{2}$ units to the right, and translated 3 units up.



Lesson 1.3, pp. 23–25

1. Answers may vary. For example, domain because most of the parent functions have all real numbers as a domain.

2. Answers may vary. For example, the end behaviour because the only two that match are x^2 and $|x|$.

3. Given the horizontal asymptote, the function must be derived from 2^x. But the asymptote is at $y = 2$, so it must have been translated up two. Therefore, the function is $f(x) = 2^x + 2$.

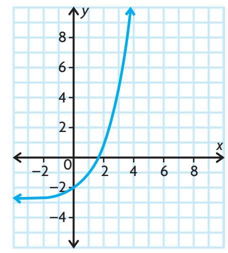
4. **a)** Both functions are odd, but their domains are different.
b) Both functions have a domain of all real numbers, but $\sin(x)$ has more zeros.
c) Both functions have a domain of all real numbers, but different end behaviour.
d) Both functions have a domain of all real numbers, but different end behaviour.

5. **a)** even **d)** odd
b) odd **e)** neither even nor odd
c) odd **f)** neither even nor odd

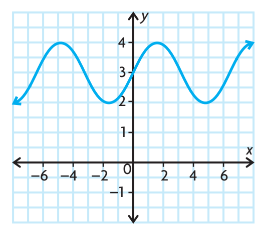
6. **a)** $|x|$, because it is a measure of distance from a number
b) $\sin(x)$, because the heights are periodic
c) 2^x, because population tends to increase exponentially
d) x, because there is \$1 on the first day, \$2 on the second, \$3 on the third, etc.

7. **a)** $f(x) = \sqrt{x}$ **c)** $f(x) = x^2$
b) $f(x) = \sin x$ **d)** $f(x) = x$

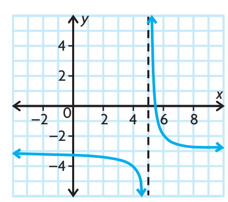
8. **a)** $f(x) = 2^x - 3$



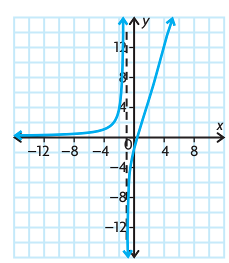
b) $g(x) = \sin x + 3$



c) $h(x) = \dfrac{1}{x-5} - 3 = \dfrac{16-3x}{x-5}$



9.



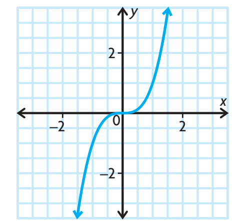
10. **a)** $f(x) = (x-2)^2$
b) There is not only one function. $f(x) = \frac{3}{4}(x-2)^2 + 1$ works as well.
c) There is more than one function that satisfies the property. $f(x) = |x-2| + 2$ and $f(x) = 2|x-2|$ both work.

11. x^2 is a smooth curve, while $|x|$ has a sharp, pointed corner at (0, 0).

12. See next page.

13. It is important to name parent functions in order to classify a wide range of functions according to similar behaviour and characteristics.

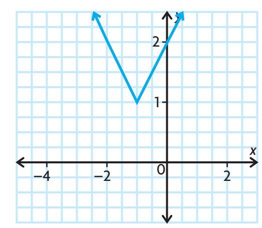
14.



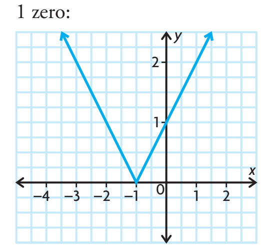
D = $\{x \in \mathbf{R}\}$, R = $\{f(x) \in \mathbf{R}\}$; interval of increase = $(-\infty, \infty)$, no interval of decrease, no discontinuities, x- and y-intercept at (0, 0), odd, $x \to \infty$, $y \to \infty$, and $x \to -\infty$, $y \to -\infty$. It is very similar to $f(x) = x$. It does not, however, have a constant slope.

15. No, $\cos x$ is a horizontal translation of $\sin x$.

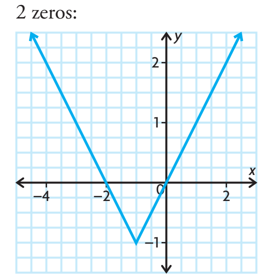
16. The graph can have 0, 1, or 2 zeros.
0 zeros:



1 zero:



2 zeros:

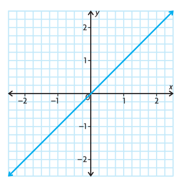
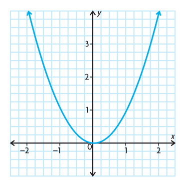
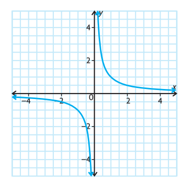
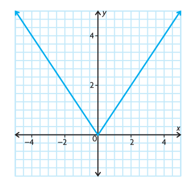
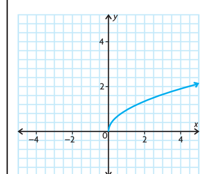
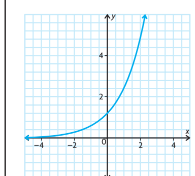
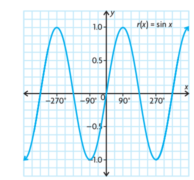


Mid-Chapter Review, p. 28

1. **a)** function; D = $\{0, 3, 15, 27\}$, R = $\{2, 3, 4\}$
b) function; D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R}\}$
c) not a function; D = $\{x \in \mathbf{R} \mid -5 \le x \le 5\}$, R = $\{y \in \mathbf{R} \mid -5 \le y \le 5\}$
d) not a function; D = $\{1, 2, 10\}$, R = $\{-1, 3, 6, 7\}$

2. **a)** Yes. Every element in the domain gets sent to exactly one element in the range.
b) D = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
c) R = $\{10, 20, 25, 30, 35, 40, 45, 50\}$

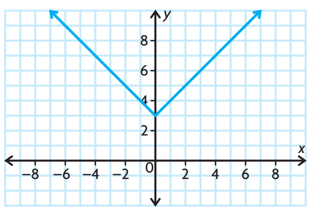
12.

Parent Function	$f(x) = x$	$g(x) = x^2$	$h(x) = \frac{1}{x}$	$k(x) = \lvert x \rvert$	$m(x) = \sqrt{x}$	$p(x) = 2^x$	$r(x) = \sin x$
Sketch							
Domain	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \neq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \geq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$
Range	$\{f(x) \in \mathbf{R}\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \neq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) > 0\}$	$\{f(x) \in \mathbf{R} \mid -1 \leq f(x) \leq 1\}$
Intervals of Increase	$(-\infty, \infty)$	$(0, \infty)$	None	$(0, \infty)$	$(0, \infty)$	$(-\infty, \infty)$	$[90(4k + 1), 90(4k + 3)]$ $K \in \mathbf{Z}$
Intervals of Decrease	None	$(-\infty, 0)$	$(-\infty, 0)$ $(0, \infty)$	$(-\infty, 0)$	None	None	$[90(4k + 3), 90(4k + 1)]$ $K \in \mathbf{Z}$
Location of Discontinuities and Asymptotes	None	None	$y = 0$ $x = 0$	None	None	$y = 0$	None
Zeros	(0, 0)	(0, 0)	None	(0, 0)	(0, 0)	None	$180k$ $K \in \mathbf{Z}$
y-Intercepts	(0, 0)	(0, 0)	None	(0, 0)	(0, 0)	(0, 1)	(0, 0)
Symmetry	Odd	Even	Odd	Even	Neither	Neither	Odd
End Behaviours	$x \to \infty, y \to \infty$ $x \to -\infty, y \to -\infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to 0$ $x \to -\infty, y \to 0$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to \infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to 0$	Oscillating

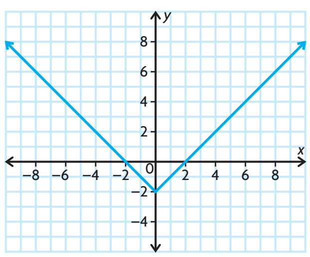
3. **a)** D $= \{x \in \mathbf{R}\}$, R $= \{f(x) \in \mathbf{R}\}$; function
b) D $= \{x \in \mathbf{R} \mid -3 \le x \le 3\}$, R $= \{y \in \mathbf{R} \mid -3 \le y \le 3\}$; not a function
c) D $= \{x \in \mathbf{R} \mid x \le 5\}$, R $= \{y \in \mathbf{R} \mid y \ge 0\}$; function
d) D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} \mid y \ge -2\}$; function

4. $-|3|, |0|, |-3|, |-4|, |5|$

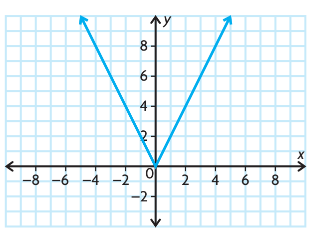
5. **a)**



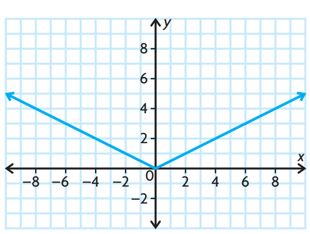
b)



c)



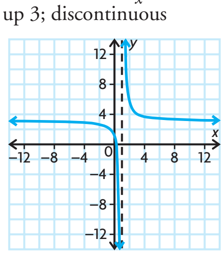
d)



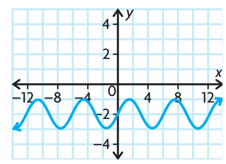
6. **a)** $f(x) = 2^x$
b) $f(x) = \dfrac{1}{x}$
c) $f(x) = \sqrt{x}$

7. **a)** even **c)** neither odd nor even
b) even **d)** neither odd nor even

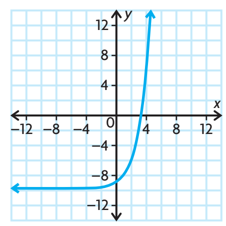
8. **a)** This is $f(x) = \frac{1}{x}$ translated right 1 and up 3; discontinuous



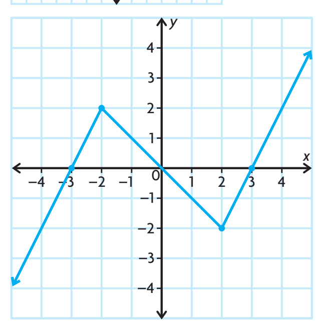
b) This if $f(x) = \sin x$ translated down 2; continuous



c) This is $f(x) = 2^x$ translated down 10; continuous



9.



Lesson 1.4, pp. 35–37

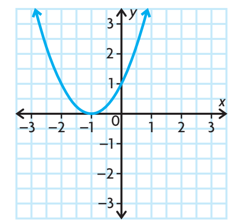
1. **a)** translation 1 unit down
b) horizontal compression by a factor of $\frac{1}{2}$, translation 1 unit right
c) reflection over the x-axis, translation 2 units up, translation 3 units right
d) reflection over the x-axis, vertical stretch by a factor of 2, horizontal compression by a factor of $\frac{1}{4}$
e) reflection over the x-axis, translation 3 units down, reflection over the y-axis, translation 2 units left
f) vertical compression by a factor of $\frac{1}{2}$, translation 6 units up, horizontal stretch by a factor of 4, translation 5 units right

2. **a)** $a = -1, k = \dfrac{1}{2}, d = 0, c = 3$
b) $a = 3, k = \dfrac{1}{2}, d = 0, c = -2$

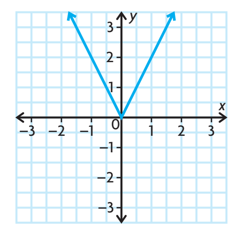
3. $(2, 3), (1, 3), (1, 6), (1, -6), (-4, -6), (-4, -10)$

4. **a)** $(2, 6), (4, 14), (-2, 10), (-4, 12)$
b) $(5, 3), (7, 7), (1, 5), (-1, 6)$
c) $(2, 5), (4, 9), (-2, 7), (-4, 8)$
d) $(1, 0), (3, 4), (-3, 2), (-5, 3)$
e) $(2, 5), (4, 6), (-2, 3), (-4, 7)$
f) $(1, 2), (2, 6), (-1, 4), (-2, 5)$

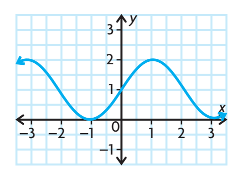
5. **a)** $f(x) = x^2$, translated left 1



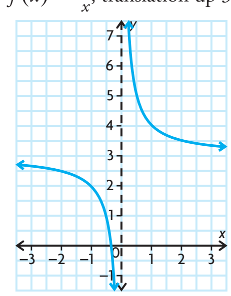
b) $f(x) = |x|$, vertical stretch by 2



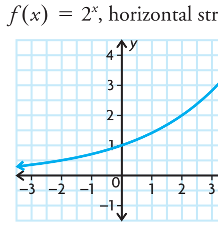
c) $f(x) = \sin x$, horizontal compression of $\frac{1}{3}$, translation up 1



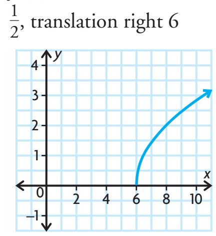
d) $f(x) = \frac{1}{x}$, translation up 3



e) $f(x) = 2^x$, horizontal stretch by 2

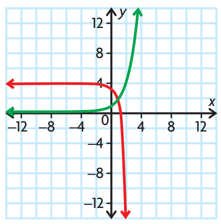


f) $f(x) = \sqrt{x}$, horizontal compression by $\frac{1}{2}$, translation right 6



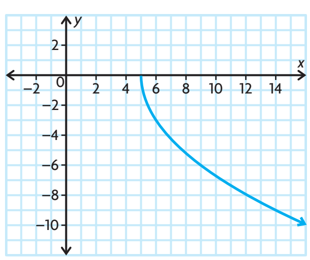
6. a) D = $\{x \in \mathbf{R}\}$,
R = $\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$
b) D = $\{x \in \mathbf{R}\}$,
R = $\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$
c) D = $\{x \in \mathbf{R}\}$,
R = $\{f(x) \in \mathbf{R} \mid 0 \leq f(x) \leq 2\}$
d) D = $\{x \in \mathbf{R} \mid x \neq 0\}$,
R = $\{f(x) \in \mathbf{R} \mid f(x) \neq 3\}$
e) D = $\{x \in \mathbf{R}\}$,
R = $\{f(x) \in \mathbf{R} \mid f(x) > 0\}$
f) D = $\{x \in \mathbf{R} \mid x \geq 6\}$,
R = $\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$

7. a)



b) The domain remains unchanged at D = $\{x \in \mathbf{R}\}$. The range must now be less than 4: R = $\{f(x) \in \mathbf{R} \mid f(x) < 4\}$. It changes from increasing on $(-\infty, \infty)$ to decreasing on $(-\infty, \infty)$. The end behaviour becomes as $x \to -\infty$, $y \to 4$, and as $x \to \infty$, $y \to -\infty$.
c) $g(x) = -2(2^{3(x-1)} + 4)$

8. $y = -3\sqrt{x - 5}$

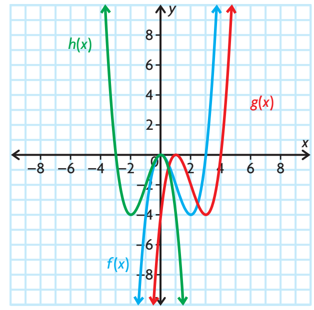


9. a) (3, 24)
b) $(-0.5, 4)$
c) $(-1, 9)$
d) $(-0.75, -8)$
e) $(-1, -8)$
f) $(-1, 7)$

10. a) D = $\{x \in \mathbf{R} \mid x \geq 2\}$,
R = $\{g(x) \in \mathbf{R} \mid g(x) \geq 0\}$
b) D = $\{x \in \mathbf{R} \mid x \geq 1\}$,
R = $\{h(x) \in \mathbf{R} \mid h(x) \geq 4\}$
c) D = $\{x \in \mathbf{R} \mid x \leq 0\}$,
R = $\{k(x) \in \mathbf{R} \mid k(x) \geq 1\}$
d) D = $\{x \in \mathbf{R} \mid x \geq 5\}$,
R = $\{j(x) \in \mathbf{R} \mid j(x) \geq -3\}$

11. $y = 5(x^2 - 3)$ is the same as $y = 5x^2 - 15$, not $y = 5x^2 - 3$.

12.



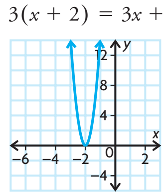
13. a) a vertical stretch by a factor of 4
b) a horizontal compression by a factor of $\frac{1}{2}$
c) $(2x)^2 = 2^2x^2 = 4x^2$

14. Answers may vary. For example:

horizontal stretch or compression, based on value of k
↓
vertical stretch or compression, based on value of a
↓
reflection in x-axis if $a < 0$; reflection in y-axis if $k < 0$
↓
horizontal translation based on value of d
↓
vertical translation based on value of c

15. (4, 5)

16. a) horizontal compression by a factor of $\frac{1}{3}$, translation 2 units to the left
b) because they are equivalent expressions: $3(x + 2) = 3x + 6$
c)



Lesson 1.5, pp. 43–45

1. a) (5, 2)
b) $(-6, -5)$
c) $(-8, 4)$
d) (2, 1)
e) $(0, -3)$
f) (7, 0)

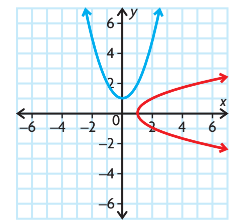
2. a) D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R}\}$
b) D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} \mid y \geq 2\}$
c) D = $\{x \in \mathbf{R} \mid x < 2\}$,
R = $\{y \in \mathbf{R} \mid y \geq -5\}$
d) D = $\{x \in \mathbf{R} \mid -5 < x < 10\}$,
R = $\{y \in \mathbf{R} \mid y < -2\}$

3. A and D match; B and F match; C and E match

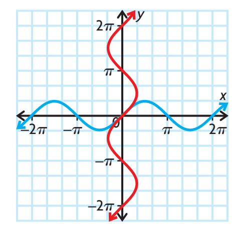
4. a) (4, 129)
b) (129, 4)
c) D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R}\}$
d) D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R}\}$
e) Yes; it passes the vertical line test.

5. a) (4, 248)
b) (248, 4)
c) D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} \mid y \geq -8\}$
d) D = $\{x \in \mathbf{R} \mid x \geq -8\}$ R = $\{y \in \mathbf{R}\}$
e) No; (248, 4) and $(248, -4)$ are both on the inverse relation.

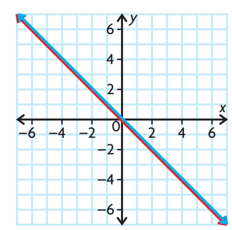
6. a) Not a function



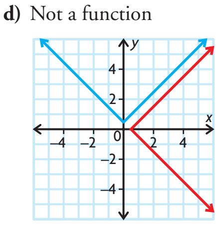
b) Not a function



c) Function



d) Not a function



7. a) $C = \frac{5}{9}(F - 32)$; this allows you to convert from Fahrenheit to Celsius.
b) 20 °C = 68 °F

8. a) $r = \sqrt{\frac{A}{\pi}}$; this can be used to determine the radius of a circle when its area is known.
b) $A = 25\pi$ cm², $r = 5$ cm

9. $k = 2$

10. a) 13
b) 25
c) 2
d) -2
e) 1
f) $\frac{1}{2}$

Answers

11. No; several students could have the same grade point average.

12. **a)** $f^{-1}(x) = \frac{1}{3}(x - 4)$

b) $h^{-1}(x) = -x$

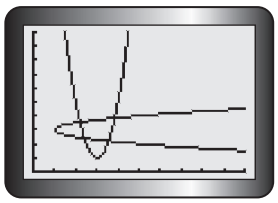
c) $g^{-1}(x) = \sqrt[3]{x + 1}$

d) $m^{-1}(x) = -\frac{x}{2} - 5$

13. **a)** $x = 4(y - 3)^2 + 1$

b) $y = \pm\sqrt{\frac{x - 1}{4}} + 3$

c)



d) (2.20, 3.55), (2.40, 2.40), (3.55, 2.20), (3.84, 3.84)

e) $x \geq 3$ because a negative square root is undefined.

f) $g(2) = 5$, but $g^{-1}(5) = 2$ or 4; the inverse is not a function if this is the domain of g.

14. For $y = -\sqrt{x + 2}$, $D = \{x \in \mathbf{R} \mid x \geq -2\}$ and $R = \{y \in \mathbf{R} \mid y \leq 0\}$. For $y = x^2 - 2$, $D = \{x \in \mathbf{R}\}$ and $R = \{y \in \mathbf{R} \mid y \geq -2\}$. The student would be correct if the domain of $y = x^2 - 2$ is restricted to $D = \{x \in \mathbf{R} \mid x \leq 0\}$.

15. Yes; the inverse of $y = \sqrt{x + 2}$ is $y = x^2 - 2$ so long as the domain of this second function is restricted to $D = \{x \in \mathbf{R} \mid x \geq 0\}$.

16. John is correct.

Algebraic: $y = \frac{x^3}{4} + 2$; $y - 2 = \frac{x^3}{4}$; $4(y - 2) = x^3$; $x = \sqrt[3]{4(y - 2)}$.

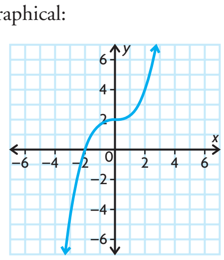
Numeric: Let $x = 4$.

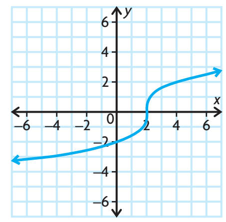
$y = \frac{4^3}{4} + 2 = \frac{64}{4} + 2 = 16 + 2 = 18$;

$x = \sqrt[3]{4(y - 2)} = \sqrt[3]{4(18 - 2)}$

$= \sqrt[3]{4(16)} = \sqrt[3]{64} = 4$.

Graphical:





The graphs are reflections over the line $y = x$.

17. $f(x) = k - x$ works for all $k \in \mathbf{R}$.

$y = k - x$

Switch variables and solve for y: $x = k - y$

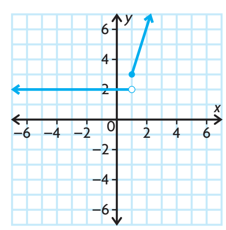
$y = k - x$

So the function is its own inverse.

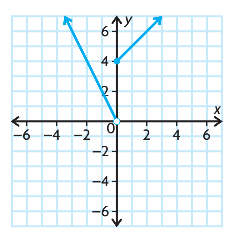
18. If a horizontal line hits the function in two locations, that means there are two points with equal y-values and different x-values. When the function is reflected over the line $y = x$ to find the inverse relation, those two points become points with equal x-values and different y-values, thus violating the definition of a function.

Lesson 1.6, pp. 51–53

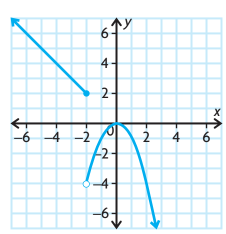
1. **a)**



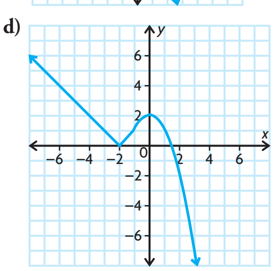
b)



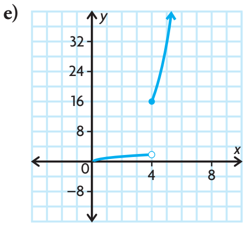
c)



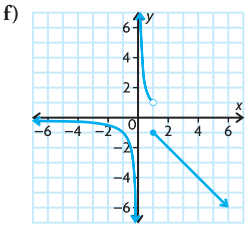
d)



e)



f)



2. **a)** Discontinuous at $x = 1$

b) Discontinuous at $x = 0$

c) Discontinuous at $x = -2$

d) Continuous

e) Discontinuous at $x = 4$

f) Discontinuous at $x = 1$ and $x = 0$

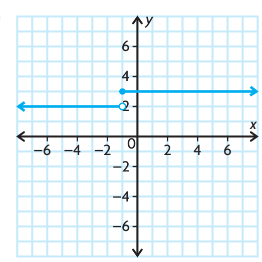
3. **a)** $f(x) = \begin{cases} x^2 - 2, \text{ if } x \leq 1 \\ x + 1, \text{ if } x > 1 \end{cases}$

b) $f(x) = \begin{cases} |x|, \text{ if } x < 1 \\ \sqrt{x}, \text{ if } x \geq 1 \end{cases}$

4. **a)** $D = \{x \in \mathbf{R}\}$; the function is discontinuous at $x = 1$.

b) $D = \{x \in \mathbf{R}\}$; the function is continuous.

5. **a)**

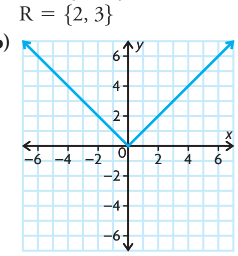


The function is discontinuous at $x = -1$.

$D = \{x \in \mathbf{R}\}$

$R = \{2, 3\}$

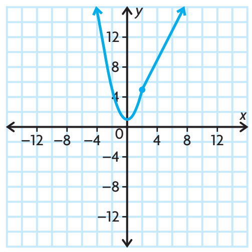
b)



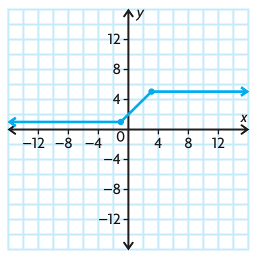
The function is continuous.

$D = \{x \in \mathbf{R}\}$

$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$

c) 

The function is continuous.
D $= \{x \in \mathbf{R}\}$
R $= \{f(x) \in \mathbf{R} \mid f(x) \geq 1\}$

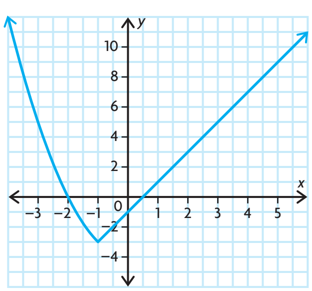
d) 

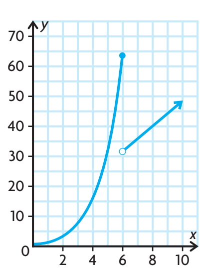
The function is continuous.
D $= \{x \in \mathbf{R}\}$
R $= \{f(x) \in \mathbf{R} \mid 1 \leq f(x) \leq 5\}$

6. $f(x) = \begin{cases} 15, \text{ if } 0 \leq x \leq 500 \\ 15 + 0.02x, \text{ if } x \geq 500 \end{cases}$

7. $f(x) =$
$$\begin{cases} 0.35x, \text{ if } 0 \leq x \leq 100\,000 \\ 0.45x - 10\,000, \text{ if } 100\,000 < x \leq 500\,000 \\ 0.55x - 60\,000, \text{ if } x > 500\,000 \end{cases}$$

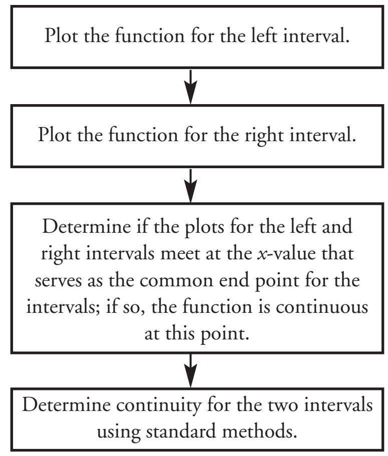
8. $k = 4$



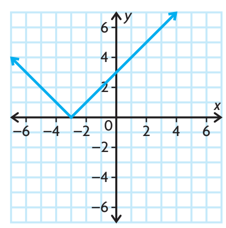
9. a) 

b) The function is discontinuous at $x = 6$.
c) 32 fish
d) $4x + 8 = 64$; $4x = 56$; $x = 14$
e) Answers may vary. For example, three possible events are environmental changes, introduction of a new predator, and increased fishing.

10. Answers may vary. For example:

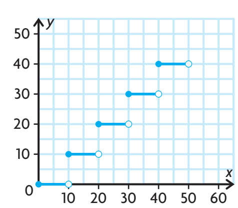


11. $f(x) = |x + 3| = \begin{cases} x + 3, \text{ if } x \geq -3 \\ -x - 3, \text{ if } x < -3 \end{cases}$



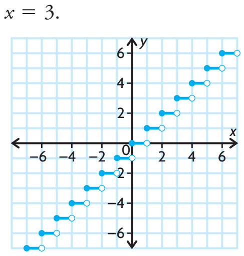
12. discontinuous at $p = 0$ and $p = 15$; continuous at $0 < p < 15$ and $p > 15$

13. $f(x) = \begin{cases} 0, \text{ if } \;\; 0 \leq x < 10 \\ 10, \text{ if } 10 \leq x < 20 \\ 20, \text{ if } 20 \leq x < 30 \\ 30, \text{ if } 30 \leq x < 40 \\ 40, \text{ if } 40 \leq x < 50 \end{cases}$



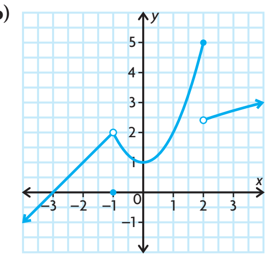
It is often referred to as a step function because the graph looks like steps.

14. To make the first two pieces continuous, $5(-1) = -1 + k$, so $k = -4$. But if $k = -4$, the graph is discontinuous at $x = 3$.

15. 

16. Answers may vary. For example:

a) $f(x) = \begin{cases} x + 3, \text{ if } x < -1 \\ x^2 + 1, \text{ if } -1 \leq x \leq 2 \\ \sqrt{x} + 1, \text{ if } x > 2 \end{cases}$

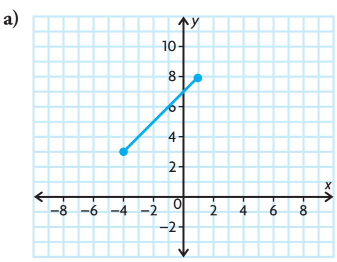
b) 

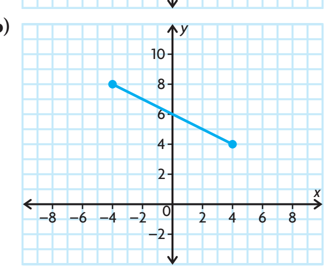
c) The function is not continuous. The last two pieces do not have the same value for $x = 2$.

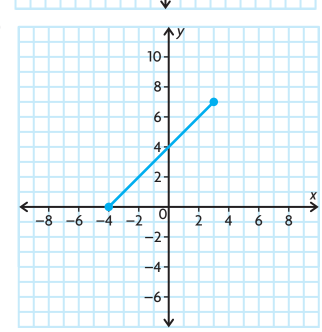
d) $f(x) = \begin{cases} x + 3, \text{ if } x < -1 \\ x^2 + 1, \text{ if } -1 \leq x \leq 1 \\ \sqrt{x} + 1, \text{ if } x > 1 \end{cases}$

Lesson 1.7, pp. 56–57

1. **a)** $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
b) $\{(-4, 2), (-2, 3), (1, 1), (4, 2)\}$
c) $\{(-4, -2), (-2, -3), (1, -1), (4, -2)\}$
d) $\{(-4, 8), (-2, 4), (1, 6), (4, 24)\}$

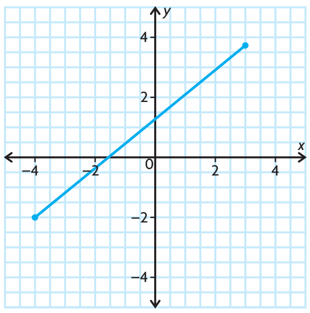
2. **a)** 

b) 

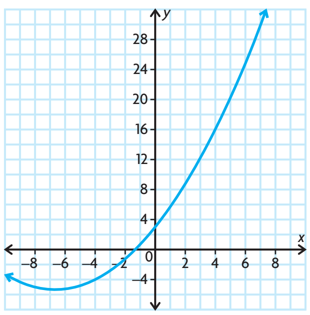
3. **a)** 

Answers

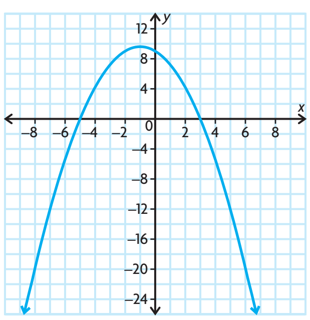
b)



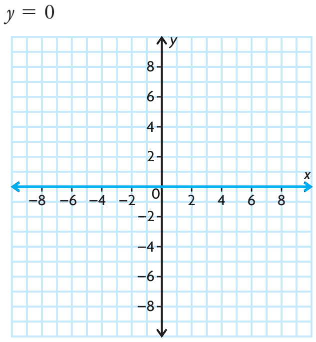
4. a)



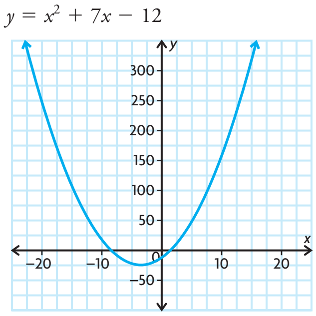
b)



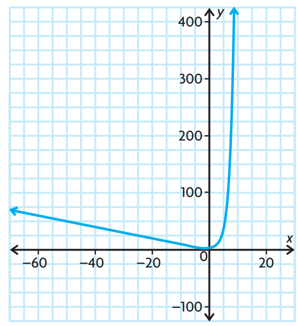
5. a) $y = 0$



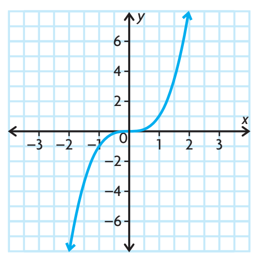
b) $y = x^2 + 7x - 12$



c) $y = |x| + 2^x$

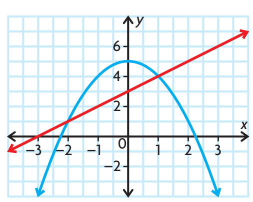


d) $y = x^3$



6. a)–b) Answers may vary. For example, properties of the original graphs such as intercepts and sign at various values of the independent variable figure prominently in the shape of the new function.

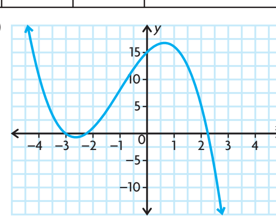
7. a)



b)

x	$f(x)$	$g(x)$	$h(x) = f(x) \times g(x)$
−3	0	−4	0
−2	1	1	1
−1	2	4	8
0	3	5	15
1	4	4	16
2	5	1	5
3	6	−4	−24

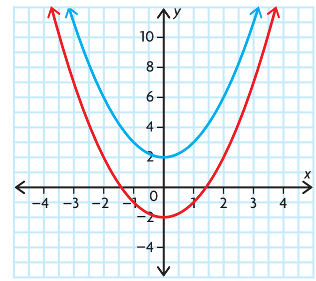
c)



d) $h(x) = (x + 3)(-x^2 + 5)$
$= -x^3 - 3x^2 + 5x + 15$; degree is 3

e) $D = \{x \in \mathbf{R}\}$; this is the same as the domain of both f and g.

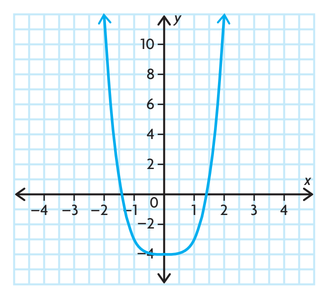
8. a)



b)

x	$f(x)$	$g(x)$	$h(x) = f(x) \times g(x)$
−3	11	7	77
−2	6	2	12
−1	3	−1	−3
0	2	−2	−4
1	3	−1	−3
2	6	2	12
3	11	7	77

c)



d) $h(x) = (x^2 + 2)(x^2 - 2) = x^4 - 4$; degree is 4

e) $D = \{x \in \mathbf{R}\}$

Chapter Review, pp. 60–61

1. a) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

b) function; $D = \{x \in \mathbf{R}\}$;
$R = \{y \in \mathbf{R} \mid y \leq 3\}$

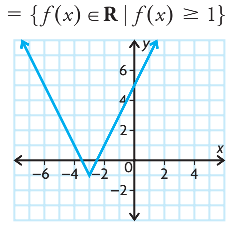
c) not a function;
$D = \{x \in \mathbf{R} \mid -1 \leq x \leq 1\}$;
$R = \{y \in \mathbf{R}\}$

d) function; $D = \{x \in \mathbf{R} \mid x > 0\}$;
$R = \{y \in \mathbf{R}\}$

2. a) $C(t) = 30 + 0.02t$

b) $D = \{t \in \mathbf{R} \mid t \geq 0\}$,
$R = \{C(t) \in \mathbf{R} \mid C(t) \geq 30\}$

3. $D = \{x \in \mathbf{R}\}$,
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 1\}$

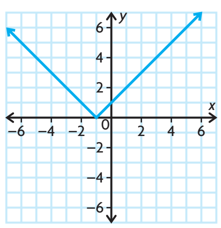


4. $|x| < 2$

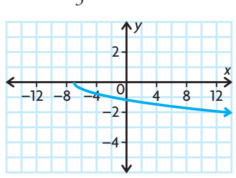
5. a) Both functions have a domain of all real numbers, but the ranges differ.
   b) Both functions are odd but have different domains.
   c) Both functions have the same domain and range, but x^2 is smooth and $|x|$ has a sharp corner at (0, 0).
   d) Both functions are increasing on the entire real line, but 2^x has a horizontal asymptote while x does not.

6. a) Increasing on $(-\infty, \infty)$; odd; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R}\}$
   b) Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$; even; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} \mid f(x) \geq 2\}$
   c) Increasing on $(-\infty, \infty)$; neither even nor odd; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} \mid f(x) > -1\}$

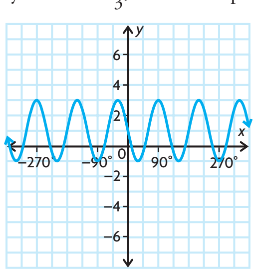
7. a) Parent: $y = |x|$; translated left 1



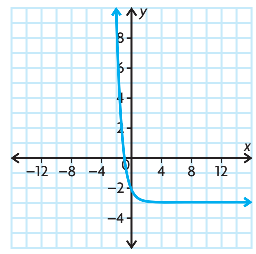
   b) Parent: $y = \sqrt{x}$; compressed vertically by a factor of 0.25, reflected across the x-axis, compressed horizontally by a factor of $\frac{1}{3}$, and translated left 7



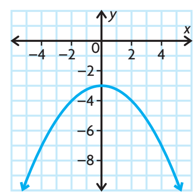
   c) Parent: $y = \sin x$; reflected across the x-axis, expanded vertically by a factor of 2, compressed horizontally by a factor of $\frac{1}{3}$, translated up by 1



   d) Parent: $y = 2^x$; reflected across the y-axis, compressed horizontally by a factor of $\frac{1}{2}$, and translated down by 3



8. $y = -\left(\frac{1}{2}x\right)^2 - 3$

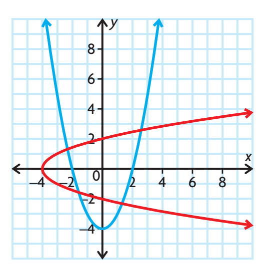


9. a) $(-2, 1)$
   b) $(-10, -6)$
   c) $(4, 3)$
   d) $\left(\frac{17}{5}, 0.3\right)$
   e) $(-1, 0)$
   f) $(9, -1)$

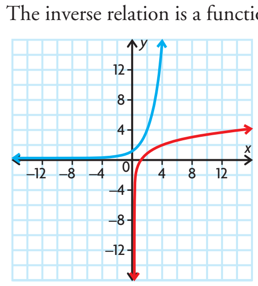
10. a) $(2, 1)$
   b) $(-9, -1)$
   c) $(7, 0)$
   d) $(7, 5)$
   e) $(-3, 0)$
   f) $(10, 1)$

11. a) $D = \{x \in \mathbf{R} \mid -2 < x < 2\}$, $R = \{y \in \mathbf{R}\}$
   b) $D = \{x \in \mathbf{R} \mid x < 12\}$, $R = \{y \in \mathbf{R} \mid y \geq 7\}$

12. a) The inverse relation is not a function.

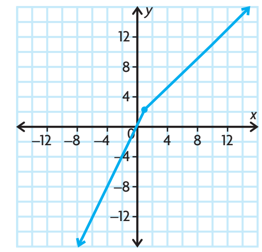


   b) The inverse relation is a function.



13. a) $f^{-1}(x) = \frac{x - 1}{2}$
   b) $g^{-1}(x) = \sqrt[3]{x}$

14.



The function is continuous; $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$

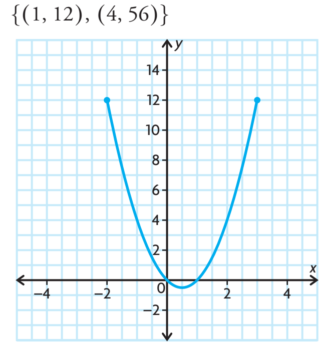
15. $f(x) = \begin{cases} 3x - 1, \text{ if } x \leq 2 \\ -x, \text{ if } x > 2 \end{cases}$; the function is discontinuous at $x = 2$.

16. In order for $f(x)$ to be continuous at $x = 1$, the two pieces must have the same value when $x = 1$. When $x = 1$, $x^2 + 1 = 2$ and $3x = 3$. The two pieces are not equal when $x = 1$, so the function is not continuous at $x = 1$.

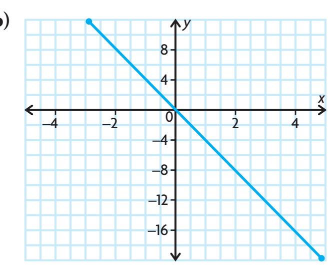
17. a) $f(x) = \begin{cases} 30, \text{ if } x \leq 200 \\ 24 + 0.03, \text{ if } x > 200 \end{cases}$
   b) \$34.50
   c) \$30

18. a) $\{(1, 7), (4, 15)\}$
   b) $\{(1, -1), (4, -1)\}$
   c) $\{(1, 12), (4, 56)\}$

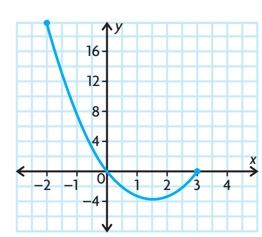
19. a)



   b)

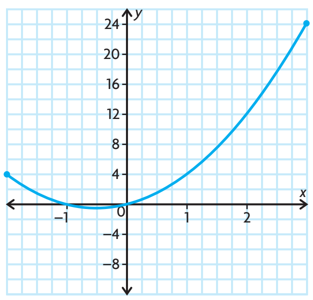


   c)

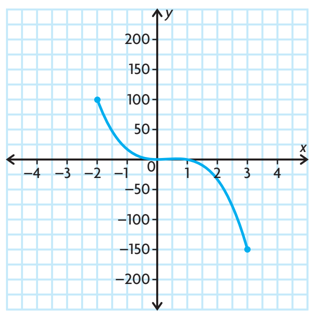


Answers

d)



e)

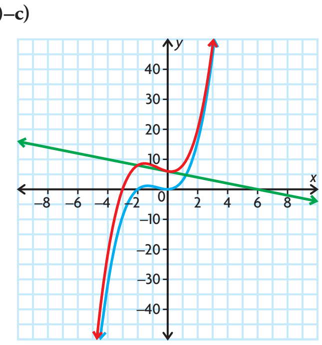


20. a) D
b) C
c) A
d) B

21. a)

x	-3	-2	-1	0	1	2
$f(x)$	-9	0	1	0	3	16
$g(x)$	9	8	7	6	5	4
$(f+g)(x)$	0	8	8	6	8	20

b)–c)



d) $x^3 + 2x^2 - x + 6$

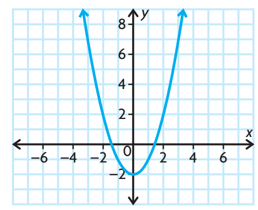
e) Answers may vary. For example, (0, 0) belongs to f, (0, 6) belongs to g and (0, 6) belongs to $f + g$. Also, (1, 3) belongs to f, (1, 5) belongs to g and (1, 8) belongs to $f + g$.

Chapter Self-Test, p. 62

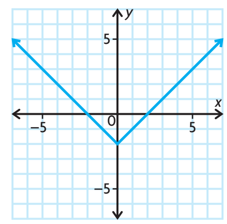
1. a) Yes. It passes the vertical line test.
b) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | y \geq 0\}$

2. a) $f(x) = x^2$ or $f(x) = |x|$

b)



or

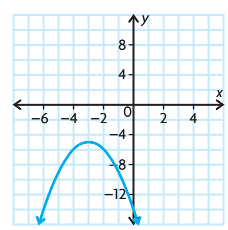


c) The graph was translated 2 units down.

3. $f(-x) = |3(-x)| + (-x)^2$
$= |3x| + x^2 = f(x)$

4. 2^x has a horizontal asymptote while x^2 does not. The range of 2^x is $\{y \in \mathbf{R} \mid y > 0\}$ while the range of x^2 is $\{y \in \mathbf{R} \mid y \geq 0\}$. 2^x is increasing on the whole real line and x^2 has an interval of decrease and an interval of increase.

5. reflection over the x-axis, translation down 5 units, translation left 3 units



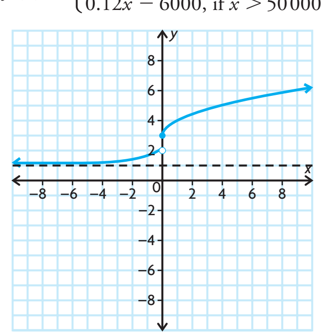
6. horizontal stretch by a factor of 2, translation 1 unit up;
$f(x) = \text{if } |\frac{1}{2}x| + 1$

7. a) $(-4, 17)$
b) $(5, 3)$

8. $f^{-1}(x) = -\frac{x}{2} - 1$

9. a) \$9000
b) $f(x) = \begin{cases} 0.05, & \text{if } x \leq 50\,000 \\ 0.12x - 6000, & \text{if } x > 50\,000 \end{cases}$

10. a)



b) $f(x)$ is discontinuous at $x = 0$ because the two pieces do not have the same value when $x = 0$. When $x = 0$, $2^x + 1 = 2$ and $\sqrt{x + 3} = 3$.
c) Intervals of increase: $(-\infty, 0)$, $(0, \infty)$; no intervals of decrease
d) $D = \{x \in \mathbf{R}\}$,
$R = \{y \in \mathbf{R} \mid 0 < y < 2 \text{ or } y \geq 3\}$

Chapter 2

Getting Started, p. 66

1. a) $\frac{4}{3}$ b) $-\frac{6}{7}$

2. a) Each successive first difference is 2 times the previous first difference. The function is exponential.
b) The second differences are all 6. The function is quadratic.

3. a) $-\frac{3}{2}, 2$ c) $45°, 225°$
b) 0 d) $-270°, -90°$

4. a) vertical compression by a factor of $\frac{1}{2}$
b) vertical stretch by a factor of 2, horizontal translation 4 units to the right
c) vertical stretch by a factor of 3, reflection across x-axis, vertical translation 7 units up
d) vertical stretch by a factor of 5, horizontal translation 3 units to the right, vertical translation 2 units down,

5. a) $A = 1000(1.08)^t$
b) \$1259.71
c) No, since the interest is compounded each year, each year you earn more interest than the previous year.

6. a) 15 m; 1 m
b) 24 s
c) 15 m

7.

Rates of Change

Linear relations	Nonlinear relations
constant; same as slope of line; positive for lines that slope up from left to right; negative for lines that slope down from left to right; 0 for horizontal lines.	variable; can be positive, negative, or 0 for different parts of the same relation

Lesson 2.1, pp. 76–78

1. a) 19 c) 13 e) 11.4
b) 15 d) 12 f) 11.04

2. a) i) 15 m/s ii) -5 m/s